Pages) **Reg. No. :** ....

ɔde No. : 5317 **Sub. Code : PHAM 24/ ZMAM 24**

.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

ıe : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

A regular vector valued function of class m is called a

(a) curve of class m

(b) path of class m

(c) differentiable function of class m

(d) function of class m

2. The point P on the curve for which $\overline{r''} = 0$ is called

(a) a singular point

(b) a central point

(c) a point of inflexion

(d) an ordinary point

3. A curve which lies on the tangent surface of C and intersects the generator orthognoally is called

(a) an evolute (b) an involute

(c) a base curve (d) an orthogonal curve

4. The osculating plane at P has ———— contact with the curve at P.

(a) four point

(b) at least four point

(c) three – point

(d) two - point

5. An ordinary point is defined as one for which rank $\begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix}$ is

(a) 1 (b) 2

(c) 0 (d) 3

For the paraboloid $x = u,\ y = v,\ z = u^2 - v^2$, E is

ˈa) $-4uv$ (b) $1+4v^2$

ˈc) $1+4u^2$ (d) u^2+v^2

Ĩhe two directions given by $Pdu^2 + 2Q\ dudv + Rdv^2 = 0$ are orthogonal on a surface, if ınd only if

a) $ER - 2FQ + GP = 0$

b) $ER + 2FQ - GP = 0$

c) $ER - 2QF - GP = 0$

d) $ER - FQ + GP = 0$

ɩ necessary and sufficient condition for a curve $\iota = u(t),\ v = v(t)$ on a surface $r = r(u,v)$ to be ːeodesic is that

a) $U\dfrac{\partial T}{\partial \dot{v}} + V\dfrac{\partial T}{\partial \dot{u}} = 0$

ɔ) $U\dfrac{\partial T}{\partial \dot{v}} - V\dfrac{\partial T}{\partial \dot{u}} = 0$

ː) $U\dfrac{\partial T}{\partial v} - V\dfrac{\partial T}{\partial u} = 0$

l) $U - V\dfrac{\partial T}{\partial \dot{u}} = 0$

9. The mean curvature μ is defined by

(a) $\mu = K_a K_b$ (b) $\mu = K_a + K_b$

(c) $2\mu = K_a + K_b$ (d) $\frac{1}{2}\mu = K_a + K_b$

10. If ϕ is the angle between the principal normal n to a curve on surface and its surface normal N, then K_n is

(a) $K\sin\phi$ (b) $K\tan\phi$

(c) $K\sec\phi$ (d) $K\cos\phi$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the arc length of one complete turn of the circular helix $r(u) = (a\cos u,\ a\sin u,\ bu)$ $-\infty < u < \infty$ where $a > 0$ and obtain the equation of the helix with s as parameter.

Or

(b) Prove that a necessary and sufficient condition for a curve to be a straight line is that $K = 0$ at all points of the curve.

12. (a) Obtain the center C and radius R spherical curvature at a point P on the given curve γ.

Or

(b) Define involute and evolute of a curve and show that the involutes of a circular helix are plane curves.

13. (a) Show that a proper parametric transformation transforms an ordinary point into an ordinary point.

Or

(b) Find E, F, G and H of the anchor ring corresponding to the domain $0 \le u \le 2\pi$, $0 \le v \le 2\pi$,

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z =$ constant.

Or

(b) Prove that the curves of the family v^3/u^2 = constant are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ $(u > 0, v > 0)$.

15. (a) Define the geodesic curvature prove that the geo desic curvature vector of any curve is orthogonal to the curve.

Or

(b) With usual notations, prove that $K_n = \dfrac{Ldu^2 + 2Mdudv + NdN^2}{Edu^2 + 2Fdudv + GdN^2}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Calculate the torsion and curvature of the cubic curve $r = (u, u^2, u^3)$.

Or

(b) State and prove Serret – Frenet formula.

17. (a) If $r = r(s)$ is the given curve γ, prove that the center C and radius R of spherical curvature at a point P on γ are given by $C = r + \rho n + \sigma p' b$, $R = \sqrt{\rho^2 + \sigma^2 p'^2}$.

Or

(b) Show that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is constant at all points.

18. (a) (i) Show that the metric is invariant under a parametric transformation.

(ii) Find angle between parametric curves.

Or

(b) If (l', m') are the direction coefficients of a line which makes an angle $\pi/2$ with the line whose direction coefficients are (l, m), then prove that $l' = -\dfrac{1}{H}(Fl + Gm)$, $m' = \dfrac{1}{H}(El + FM)$.

19. (a) (i) Prove that the two directions given by $Pdu^2 + 2Q\,dudv + Rdv^2 = 0$ are orthogonal on a surface if and only if $ER - 2QF + GP = 0$.

(ii) Also prove that if θ is the angle between the two curves, then $\tan\theta = \dfrac{2H(Q^2 - PQ)^{1/2}}{ER - 2FQ + GP}$.

Or

(b) Prove that any curve u=u(t), $v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

20. (a) State and prove Liouville's formula.

Or

(b) If K is the normal curvature in a direction making an angle ψ with the principal direction γ = constant, then prove that $K = K_a \cos^2\psi + K_b \sin^2\psi$ where K_a and K_b are principal curvatures at the point P on the surface.

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